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COMMENT

Comment on 'Eden model on the Manhattan lattice'

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Abstract. We re-examine the two-parameter position space renormalisation group calculation for Eden growth process on the square Manhattan lattice (SML), first studied by Chernoutsan and Milosevic, and find that, contrary to the finding of the above authors, Eden processes on SML and on the ordinary square lattice are in the same universality class.

Recently, Chernoutsan and Milosevic (CM) (1985) applied a two-parameter renormalisation group technique to the Eden model on the square Manhattan lattice (SML) and concluded that this model and the Eden model on the ordinary square lattice are in two different universality classes. Since the Eden process is believed to yield compact clusters with fractal dimension D_E equal to the spatial dimension d (Peters *et al* 1979), the same process on the completely connected SML is also expected to yield compact clusters with fractal dimension D_{EM} equal to the spatial dimension d. The conclusion of CM on the contrary, naturally led to some surprises. Botet (1986) later did some simulation work of Eden clusters on the SML and concluded that the Eden model on the ordinary square lattice and the Eden model on SML are very similar and probably in the same universality class. To this CM responded (1986) that the question of universality cannot be settled by numerical simulation alone. In order to shed more light on this issue, we have undertaken a re-examination of the renormalisation calculation of CM.

The renormalisation scheme is shown in figure 1, which shows the division of the SML into 3×3 cells and a renormalisation which preserves the square lattice symmetry and Manhattan orientations of the original lattice. The bond orientation is achieved

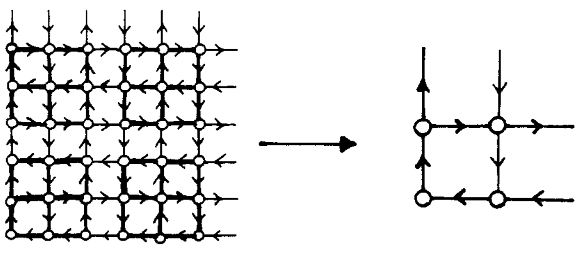

Figure 1. 3×3 cell to site rescaling of the SML; bond orientations are renormalised by using majority rule.

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by a majority rule. Following Prentis (1984) and Malakis (1984), we allow that an act of growth on the SML may violate the underlying bond orientation with a probability $(1-p)$. An act of growth obeying the underlying bond orientation, however, occurs with probability p. The fugacity for adding an extra particle is K. In order to study Eden process on SML, an extended form of the Eden model had been introduced (CM 1985), which we will refer to in what follows simply as the Eden model. Hence the weight of a l-site cluster is $K^l p^s (1-p)^r$ if the cluster has been grown through s steps that obey the underlying orientations of the bonds and through r steps that violate the underlying orientations, so that $r+s=l-1$. By enumerating the total weights F_A and F_C of all spanning clusters that can be grown on the cells A and C, we find the recursion relations

$$p'^2 K'^2 = [(1+p)/3]^2 F_A(K, p, 1-p) F_C(K, p, 1-p) \tag{1}$$

$$(1-p')^2 K'^2 = [(2-p)/3]^2 F_A(K, 1-p, p) F_C(K, 1-p, p). \tag{2}$$

The total weights F_A and F_C given in (1) and (2) are exactly the same as those in CM. Equation (2) follows from (1) by simply reversing all the bond orientations of figure 1. The factors $[(1+p)/3]^2$ and $[(2-p)/3]^2$ in (1) and (2) result from taking the average over the three possible directions of intercell bonds in each case. For comparison the recursion relations of CM are rewritten here as

$$p' K'^2 = p F_A(K, p, 1-p) F_C(K, p, 1-p) \tag{1'}$$

$$(1-p') K'^2 = (1-p) F_A(K, 1-p, p) F_C(K, 1-p, p). \tag{2'}$$

In equations (1) and (2), two consecutive steps of the bond orientation in the renormalised lattice have been used, leading to factors p'^2 and $(1-p')^2$ on the left-hand sides. On the other hand, in equations (1′) and (2′) only one step of the bond orientation in the renormalised lattice has been used. Hence the factors p' and $(1-p')$. By taking two consecutive steps of the bond orientation in the renormalised lattice we have preserved explicitly the Manhattan orientations of the original lattice. Since in our renormalisation procedure, we have a cell to site transformation for the fugacity K but a cell to bond renormalisation for the bond orientation parameter p, the use of two consecutive steps of the bond orientation in the renormalised lattice is important. The situation is different in the renormalisation of self-avoiding walks on SML (Prentis 1984) in which one has a cell to bond renormalisation for both the step fugacity and the bond orientation. For instance, the renormalised cells shown in figure 2, which

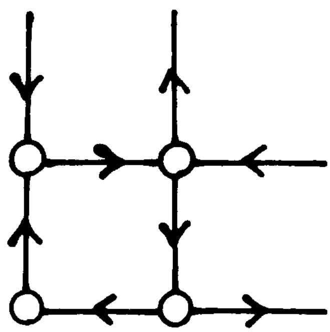

Figure 2. An example of renormalised cells not preserving Manhattan orientation but nevertheless not excluded in (1′) and (2′).

do not preserve the Manhattan orientations of the original lattice, are not excluded in the recursion relations (1′) and (2′). Also, by averaging over the three possible intercell bond orientations, we have the factors $[(1+p)/3]^2$ and $[(2-p)/3]^2$ on the right-hand sides of (1) and (2). Therefore we believe that our recursion relations (1) and (2) are a more faithful representation of the renormalisation process shown in figure 1 than (1′) and (2′) of CM.

Iterating (1) and (2) we obtain the flow diagram shown in figure 3. We find now only one non-trivial fixed point at $(K, p) = (0.463, 0.5)$ corresponding to the non-oriented problem. The critical surface is formed by the set of trajectories that flow into this non-trivial fixed point. The intersection of the critical surface with the $p = 1$ ($p = 0$) axis is a statement of the universality of the Manhattan (anti-Manhattan) problem and the ordinary square lattice problem. The intersection point determines the non-universal critical fugacity K_C characterising the Manhattan lattice problem. We find this intersection point at $K_C = 0.444$. Contrary to the result of CM, we find that the Eden model on SML and the Eden model on the square lattice belong to the same universality class. We have also tried the recursion relations

$$p'^2K'^3 = [(1+p)/3]^2F_A^2(K, p, 1-p)F_C(K, p, 1-p)$$

$$(1-p')^2K'^3 = [(2-p)/3]^2F_A^2(K, 1-p, p)F_C(K, 1-p, p)$$

but the flow pattern is qualitatively the same as in figure 3. The replacement of the factors p and $(1-p)$ on the left-hand sides of (1′) and (2′) by the more general factors $(1+p)/3$ and $(2-p)/3$ is crucial, for otherwise the points $(K, p) = (0, 1/2), (0, 1)$ would still be trivial fixed point of the transformation even if two consecutive bonds in the renormalised lattice were used and we would have the same flow pattern as in CM.

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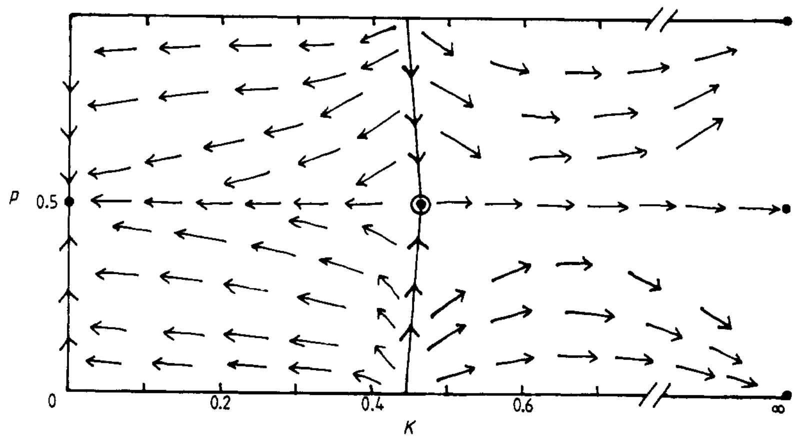

Figure 3. Flow diagram generated by recursion relations (1) and (2). The non-trivial (◉) and trivial (●) fixed points are shown. The critical surface (full curve) is shown as the set of trajectories flowing into the non-trivial fixed point. The intersection of the critical surface of the non-oriented Eden model with the $p = 1$ (Manhattan Eden model) axis is a statement of the universality of these two models.

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